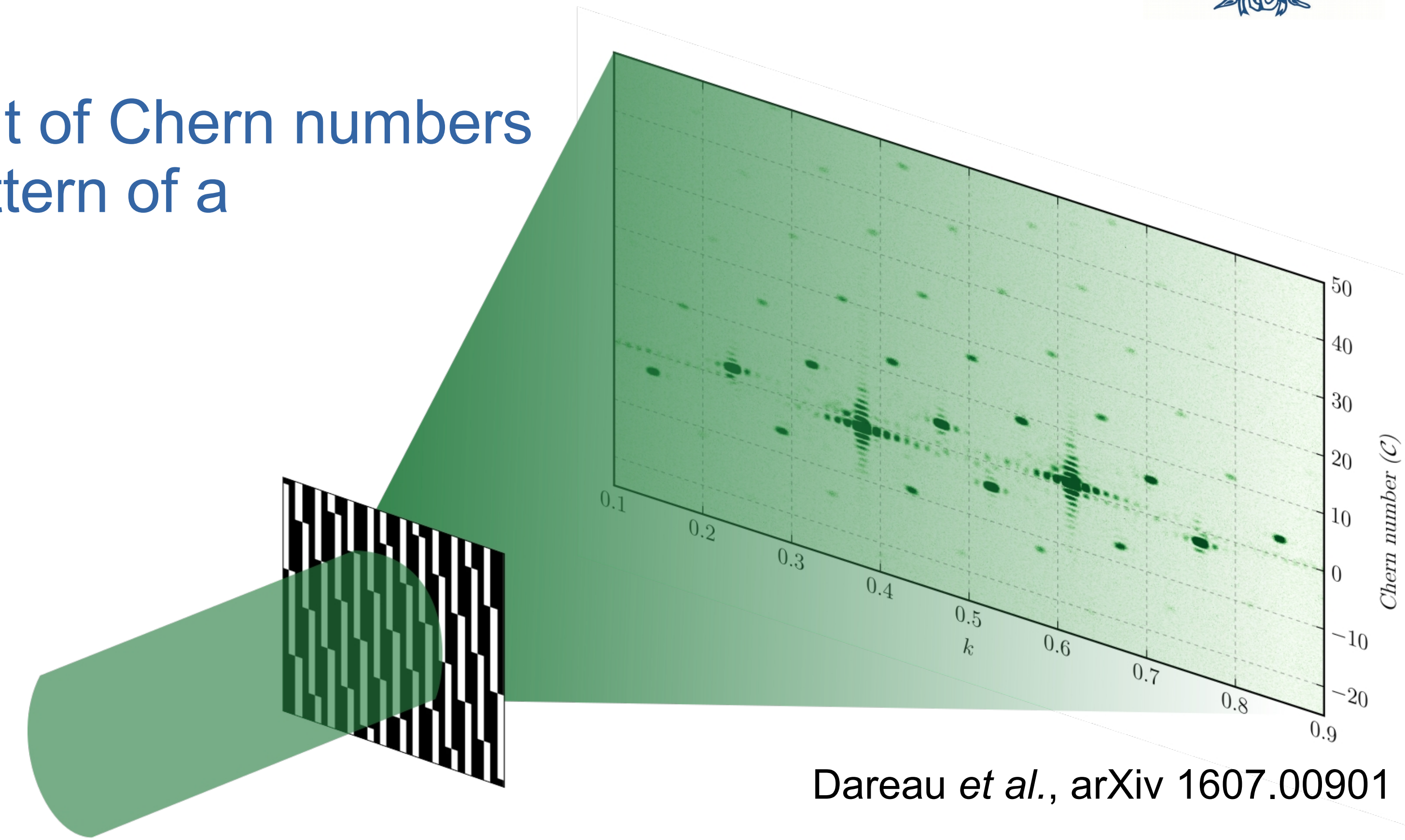
Direct measurement of Chern numbers in the diffraction pattern of a Fibonacci chain.

JMC15 – Bordeaux

25th August 2016



Dareau *et al.*, arXiv 1607.00901

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E. Akkermans (*), F. Gerbier & J. Beugnon

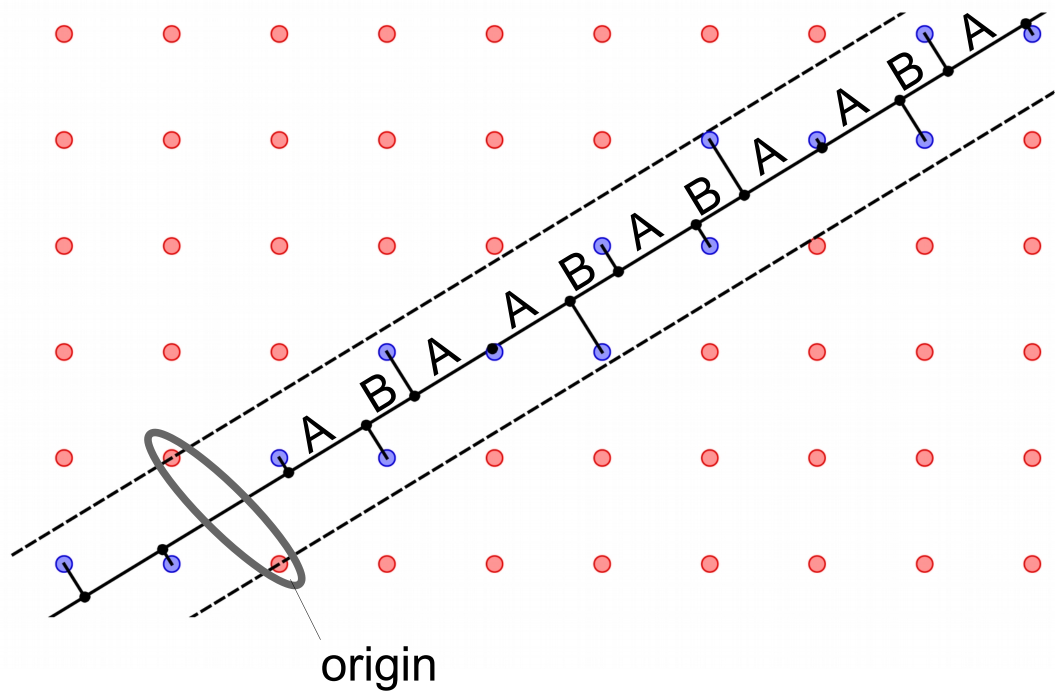
Laboratoire Kastler Brossel, Collège de France, CNRS, ENS, UPMC.
(*) Technion Israel Institute of Technology, Department of Physics (Israel)

Fibonacci Chain

Constructing the Fibonacci chain

$$\tau = \frac{1+\sqrt{5}}{2}$$

- Cut and Project (C&P)



slope = $1/\tau$

irrational

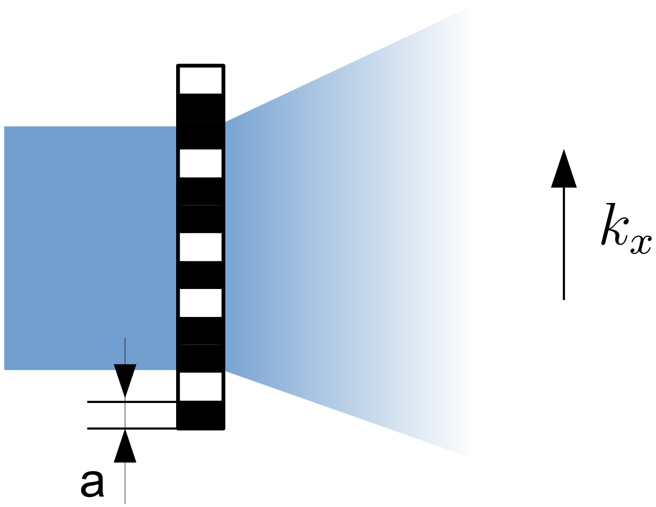
aperiodic structure (quasicrystal)

ABAABABAABA...

(in this case : Fibonacci)

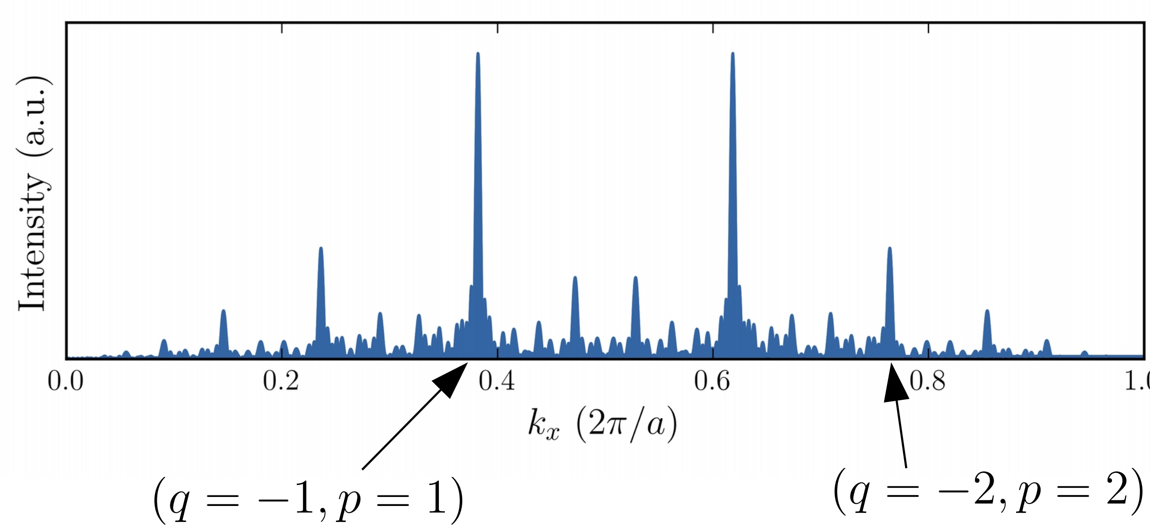
NB : aperiodic order comes from projection of a periodic structure of higher dimension

Diffraction from a Fibonacci chain



Peaks positions given by two integers

$$k_x(p,q) \propto p + \frac{q}{\tau}$$



Topological properties of the 1D Fibonacci chain

$$\tau = \frac{1+\sqrt{5}}{2}$$

- From density of states
  - → multi-gap system
  - → **gap labeling theorem** (Bellissard, 1982)

gaps position in reciprocal space : $$k_{q,p} = p + q/\tau$$

with p and q : integers

NB : gaps open at the position of the diffraction peaks.

q is a Chern number

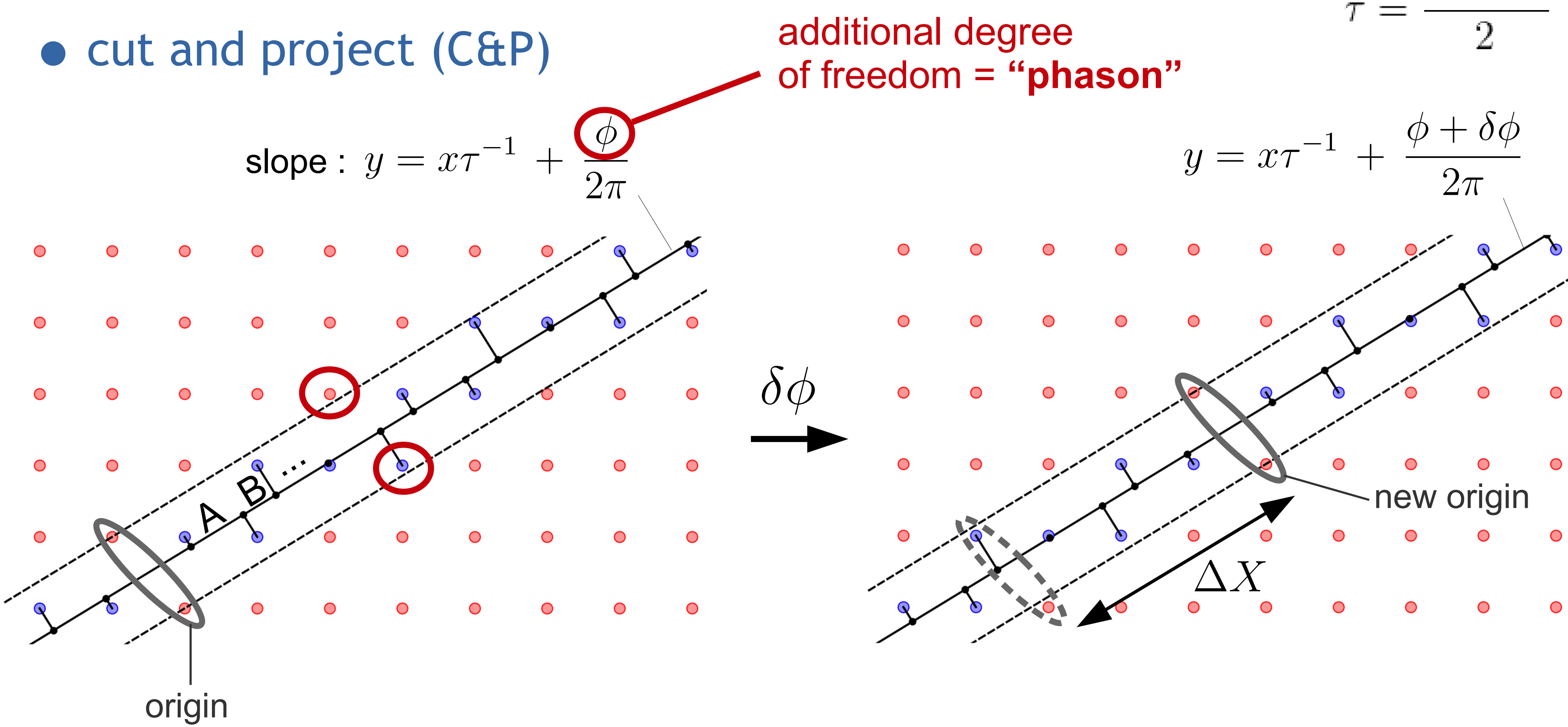
- Connected to structural properties Levy *et al.*, arXiv 1509.04028

"phason" degree of freedom

Phason degree of freedom

- cut and project (C&P)

$$\tau = \frac{1+\sqrt{5}}{2}$$



scanning phason Φ for a finite chain → spatial shift ΔX

Effect of the phason ?

- scanning Φ → spatial shift : ΔX
- spatial shift (real space) → phase shift (reciprocal space)

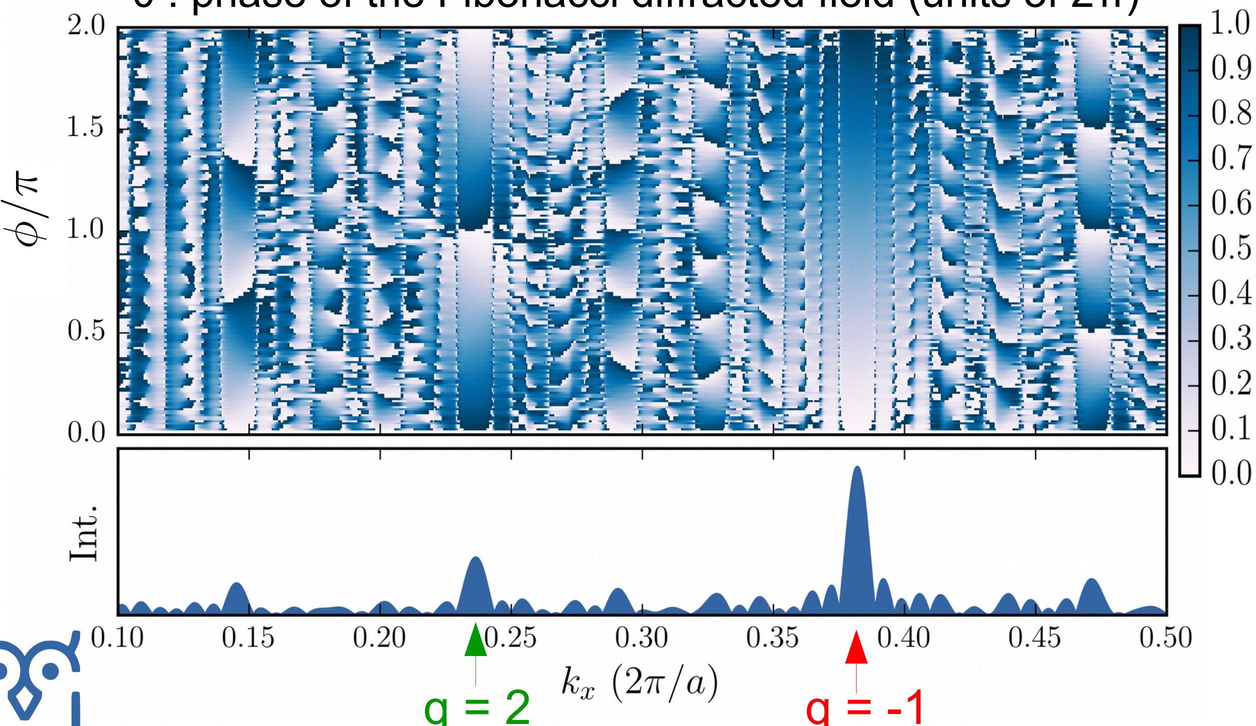
➡ **phason affects the phase** of the diffracted field

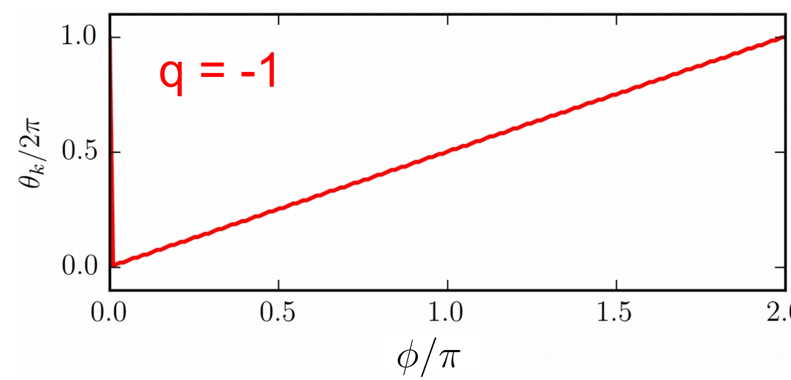
- for a diffraction peak at $k_x(p,q)$ the phase shift is $k_x(p,q)\Delta X = -q\phi\,[2\pi]$

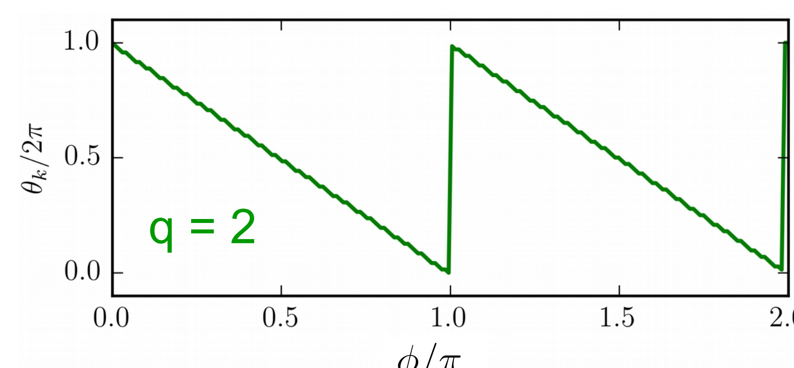
Dareau *et al.*, arXiv 1607.00901

Example : for $\mathrm{F_n} = 144$

θ : phase of the Fibonacci diffracted field (units of 2π)

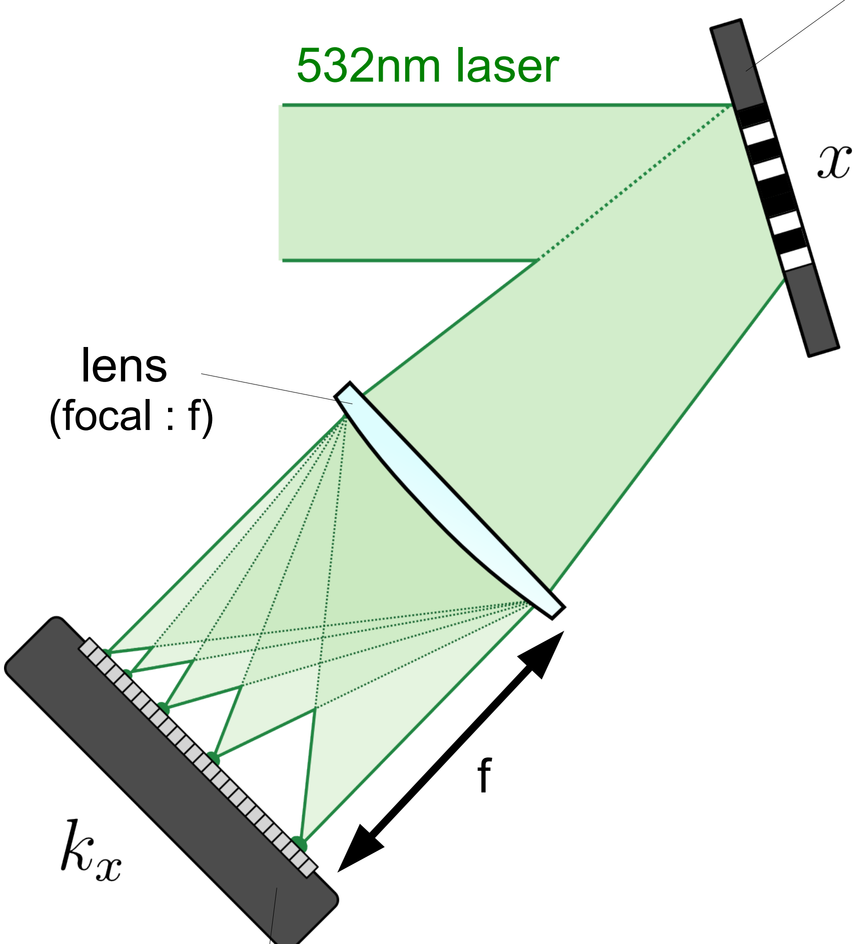




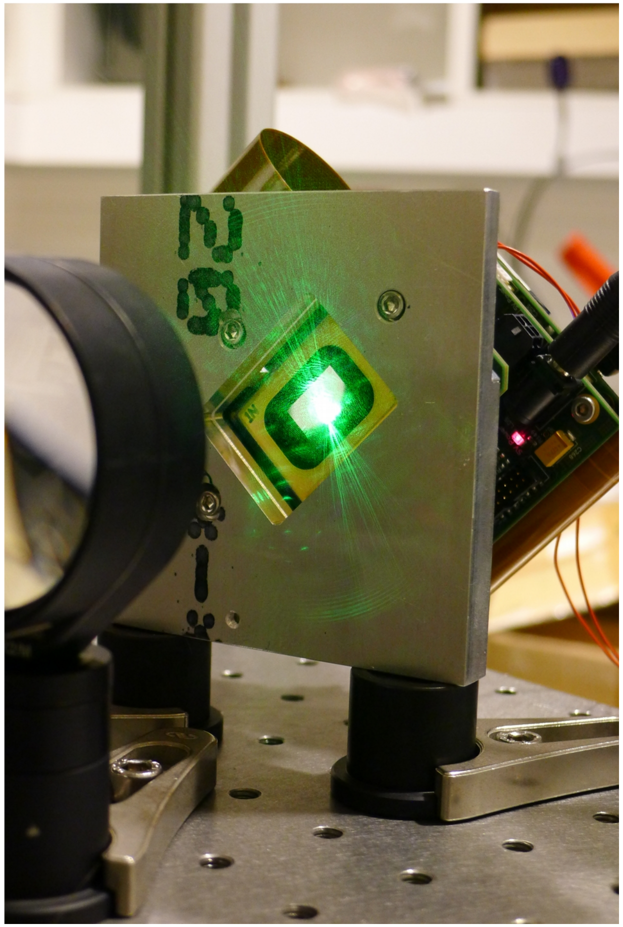


Optical diffraction by a Fibonacci chain

Our experimental setup



Digital Micromirror Device (DMD)
- mirror ("pixel") size ~ 14 μm
- 1024 × 768 pixels



CCD camera ➡ located at the Fourier plane of the DMD image

➝ **Fraunhofer (far-field) diffraction pattern**

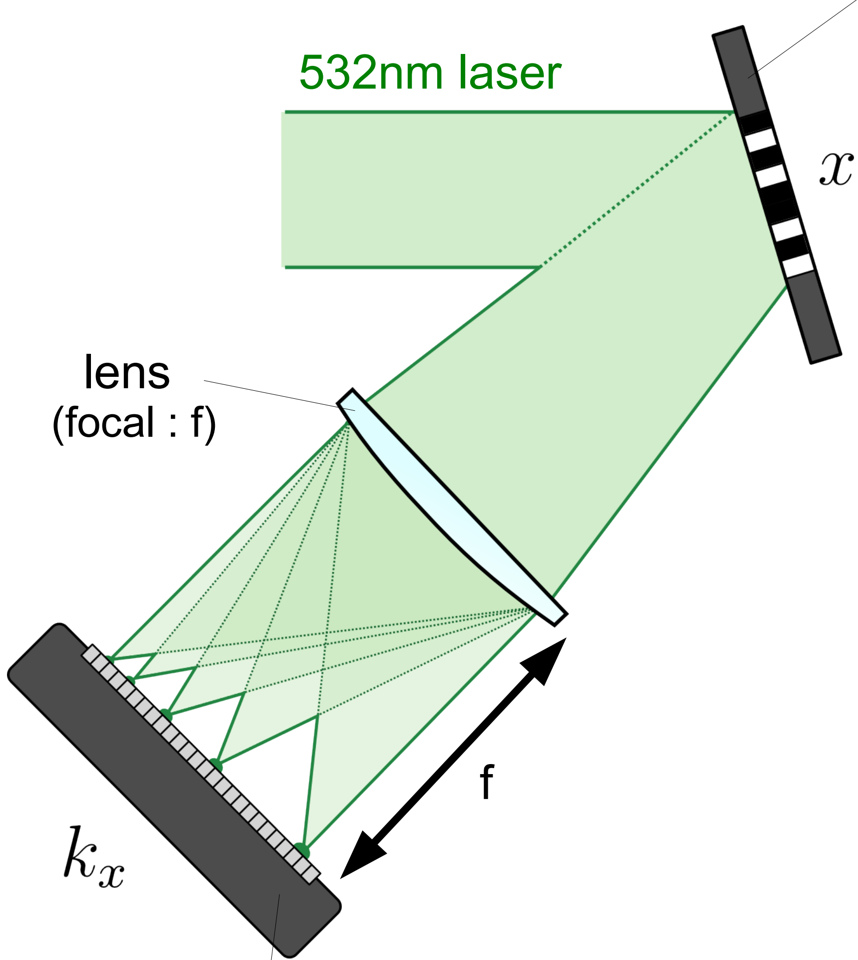
Optical diffraction by a Fibonacci chain

Our experimental setup

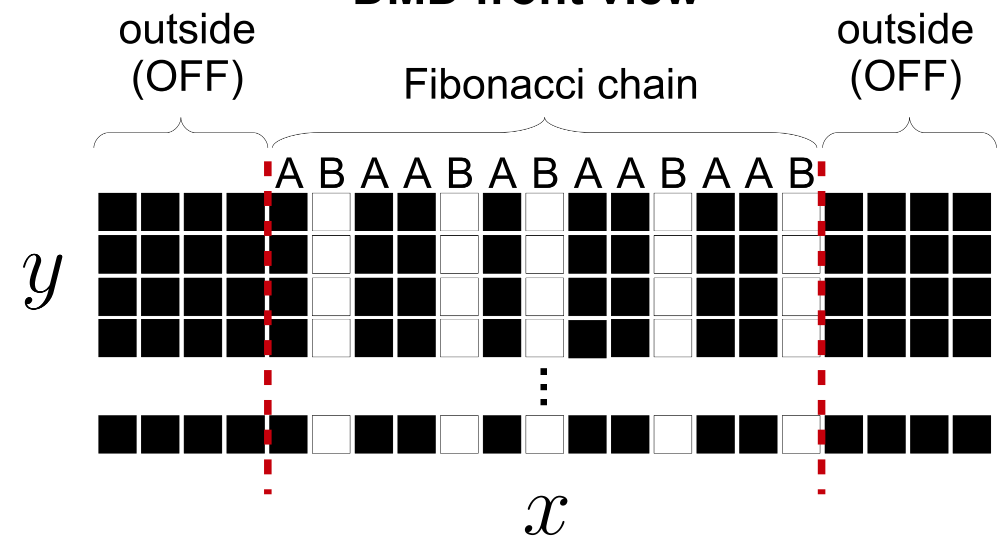
Digital Micromirror Device (DMD)

- mirror ("pixel") size ~ 14 µm
- 1024 × 768 pixels

Fibonacci encoding : A = ■ (pixel OFF)
B = □ (pixel ON)



DMD front view



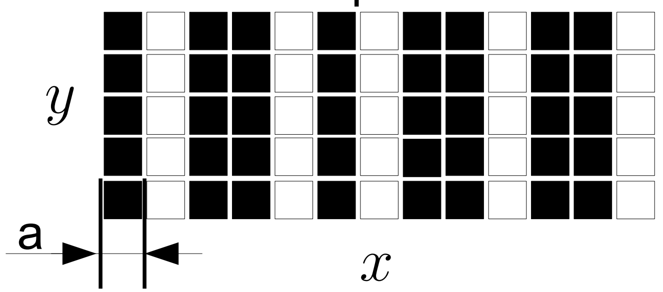
CCD camera ➡ located at the Fourier plane of the DMD image

→ **Fraunhofer (far-field) diffraction pattern**

Optical diffraction by a Fibonacci chain

Diffraction by a single Fibonacci chain

DMD pattern



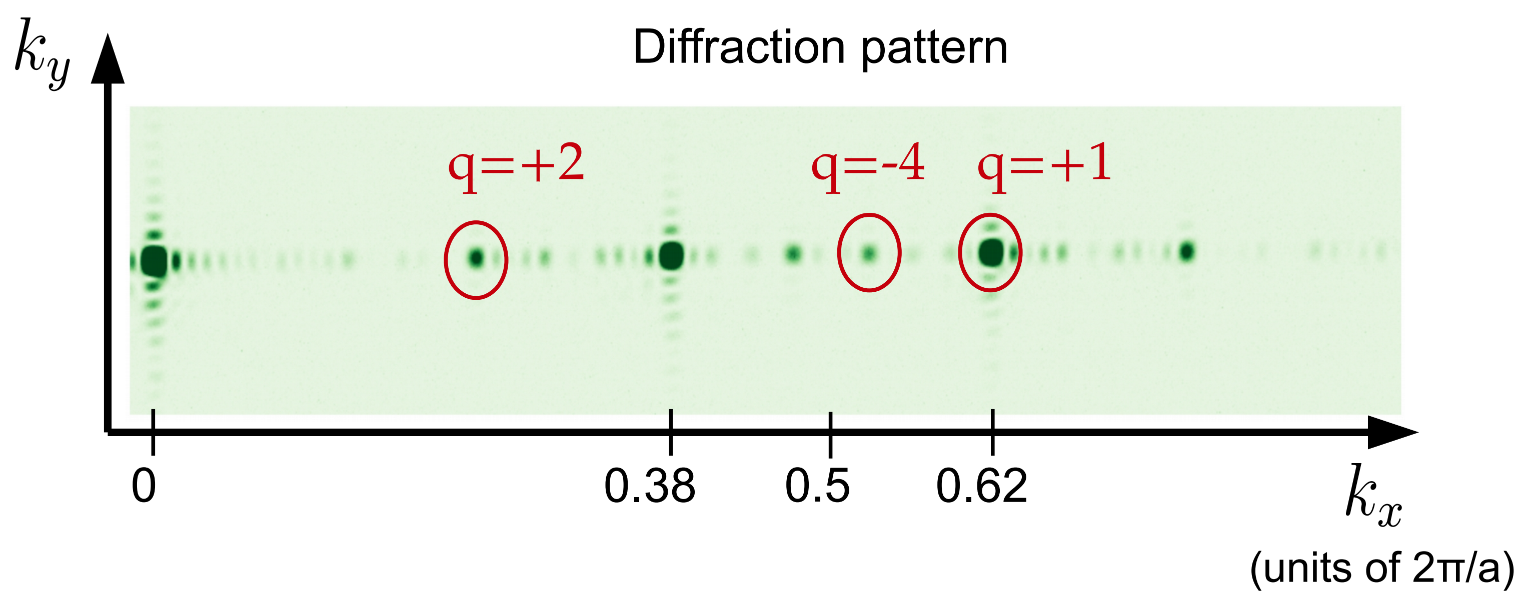
peaks located at

$$k_{q,p} = p + q/\tau$$

(in units of 2π/a)

Ex : main peaks (q=±1)

$$k_{1,0} \approx 0.62$$
$$k_{-1,1} \approx 0.38$$

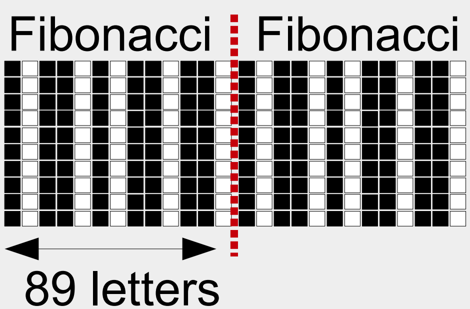


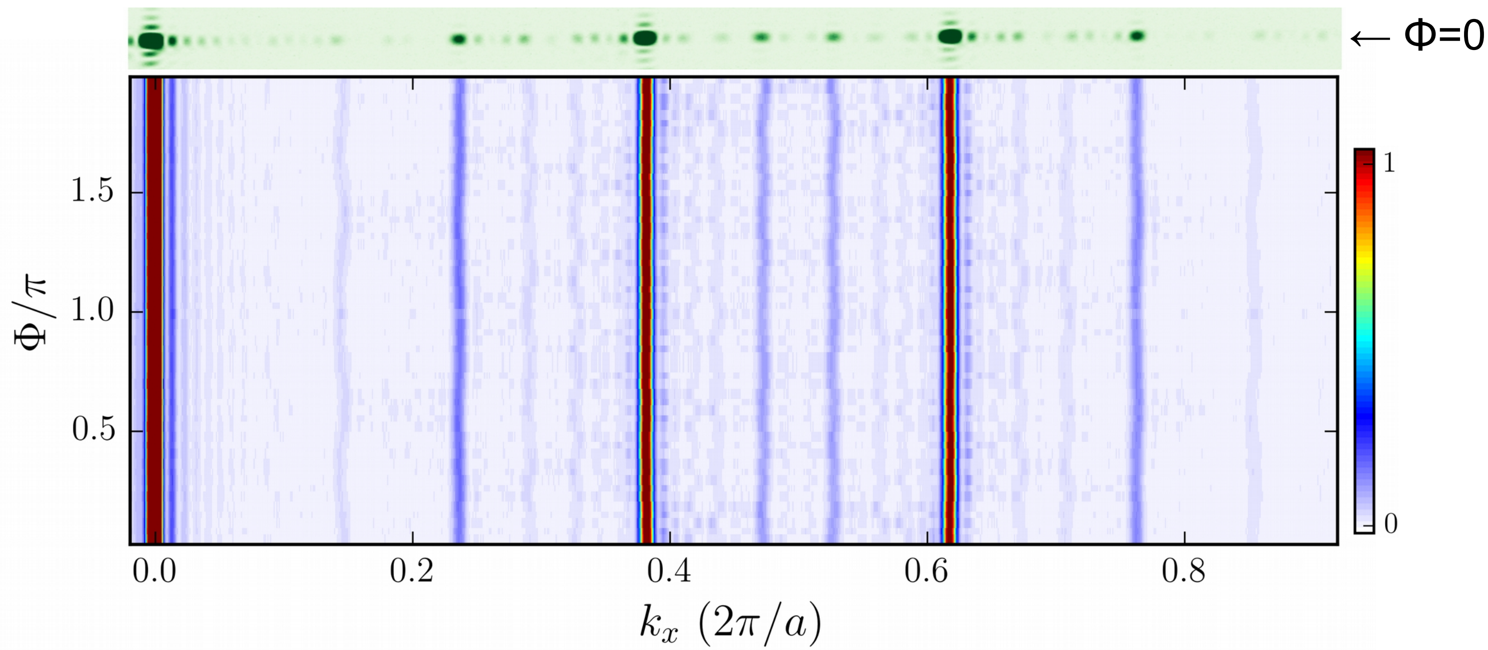
Optical diffraction by a Fibonacci chain

Scanning the phason : results

Dareau *et al.*, arXiv 1607.00901

DMD Pattern



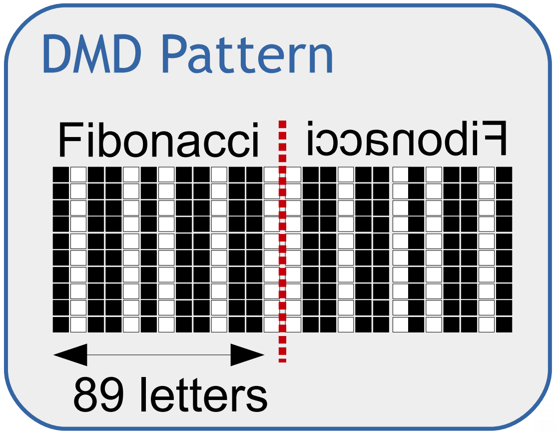


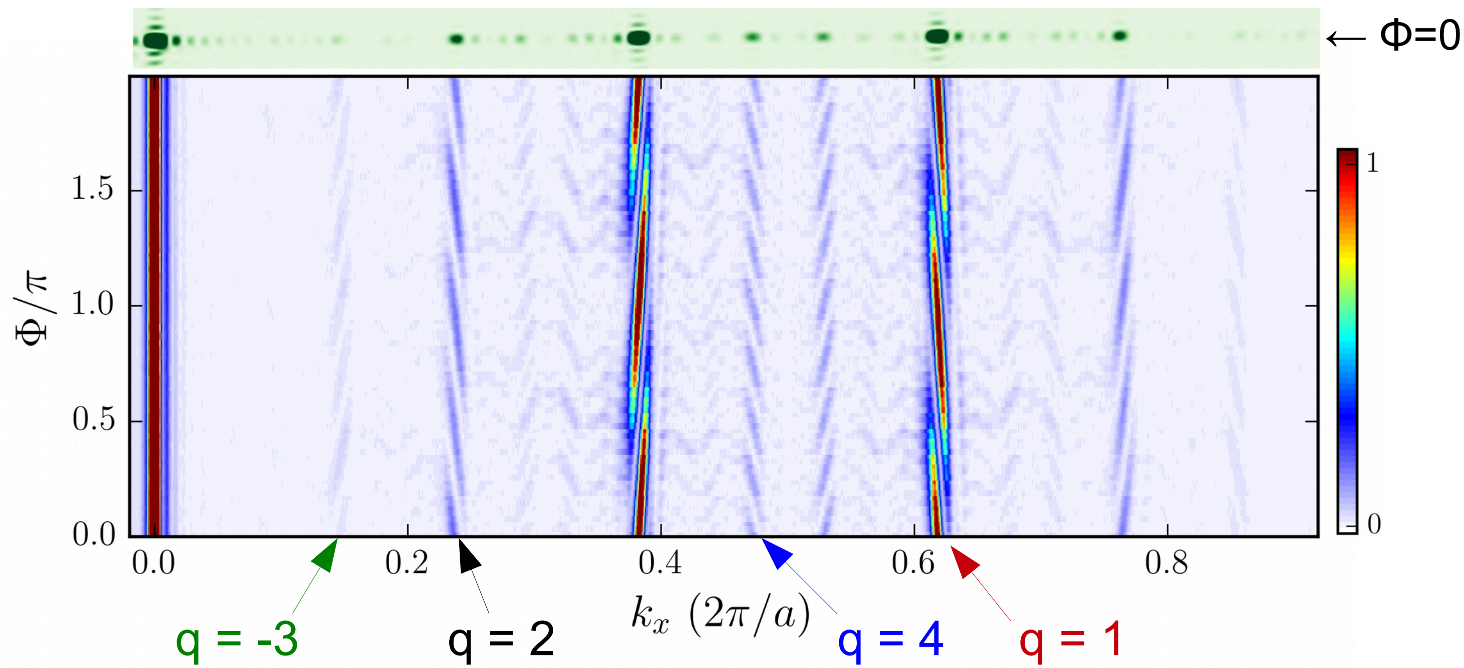
No effect of the phason scan !

Optical diffraction by a Fibonacci chain

Scanning the phason : results

Dareau *et al.*, arXiv 1607.00901





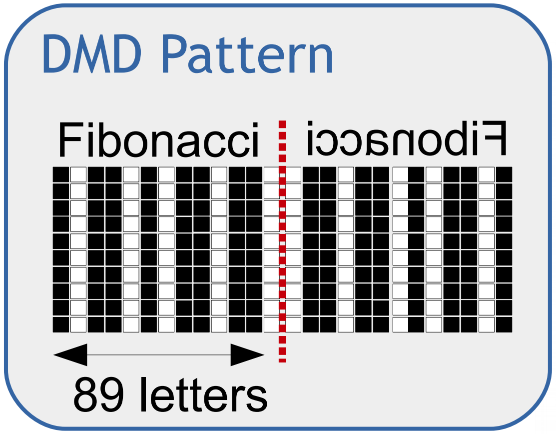
Peaks are crossed by holes

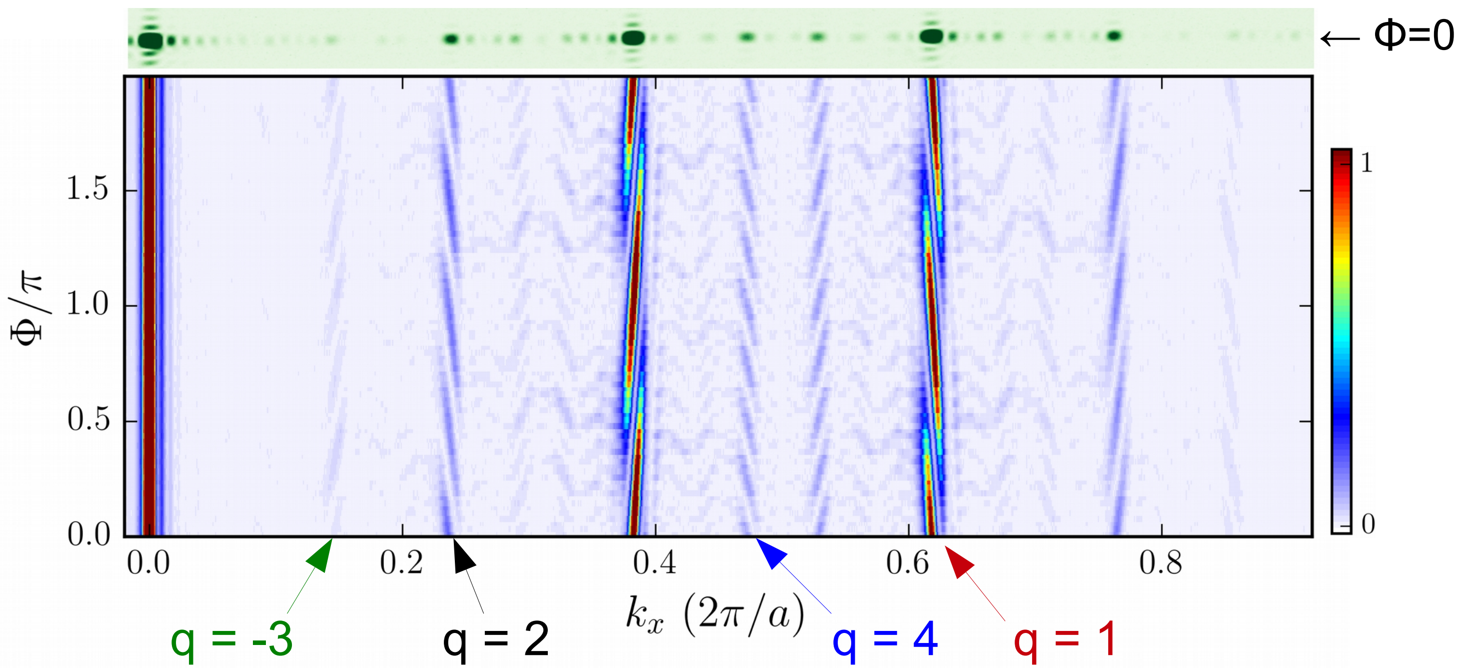
Slope / number of crossings gives the Chern number q

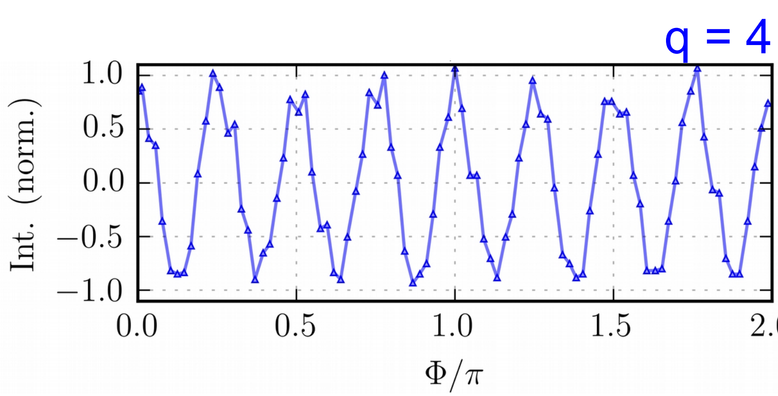
Optical diffraction by a Fibonacci chain

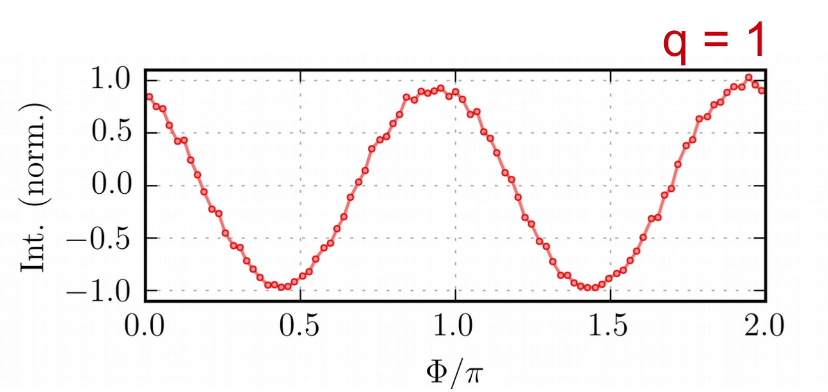
Scanning the phason : results

Dareau *et al.*, arXiv 1607.00901







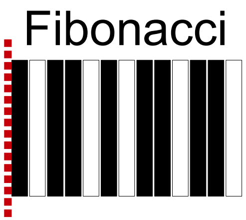


k_x cuts at initial peak position : oscillation with period π/q

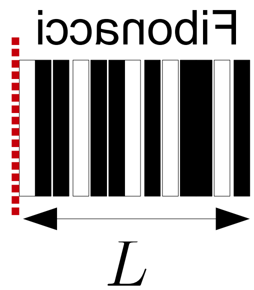
Optical diffraction by a Fibonacci chain

Scanning the phason : discussion

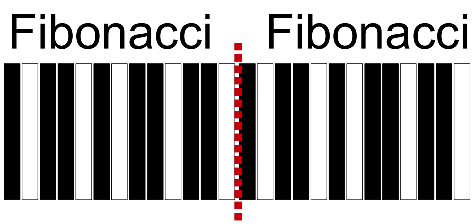
$\phi_0 = k_x L$



$$\mathcal{A}(k_x, \phi) = \mathcal{A}_0(k_x) e^{-iq\phi}$$

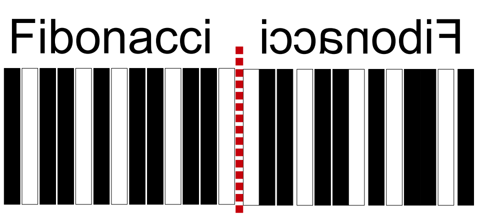


$$\mathcal{A}(k_x, \phi) = \underbrace{\mathcal{A}_0(-k_x) e^{iq\phi}}_{x \to -x} \underbrace{e^{ik_x L}}_{\text{spatial shift}}$$
$$= \mathcal{A}_0(k_x) e^{iq\phi} e^{i\phi_0}$$



$$I(k_x, \phi) = |\mathcal{A}_0(k_x)|^2 \times |e^{-iq\phi} e^{-i\phi_0} + e^{-iq\phi}|^2$$
$$= |\mathcal{A}_0(k_x)|^2 \times 4 \cos^2(\phi_0/2)$$

no Φ dependence

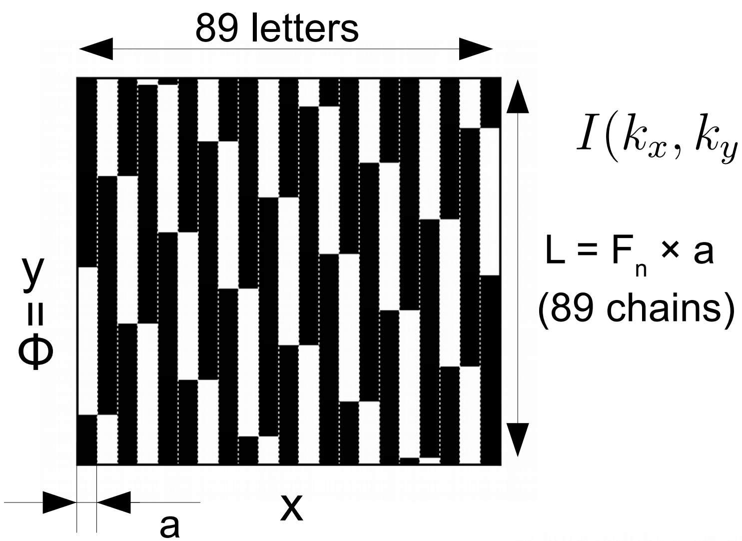


$$I(k_x, \phi) = |\mathcal{A}_0(k_x)|^2 \times |e^{-iq\phi} e^{-i\phi_0} + e^{+iq\phi} e^{+i\phi_0}|^2$$
$$= |\mathcal{A}_0(k_x)|^2 \times 4 \cos^2(q\phi - \phi_0)$$

⟶ sinusoidal variation with Φ, period T = π/q

Optical diffraction by a Fibonacci chain

Diffraction from 2D (x,Φ) pattern

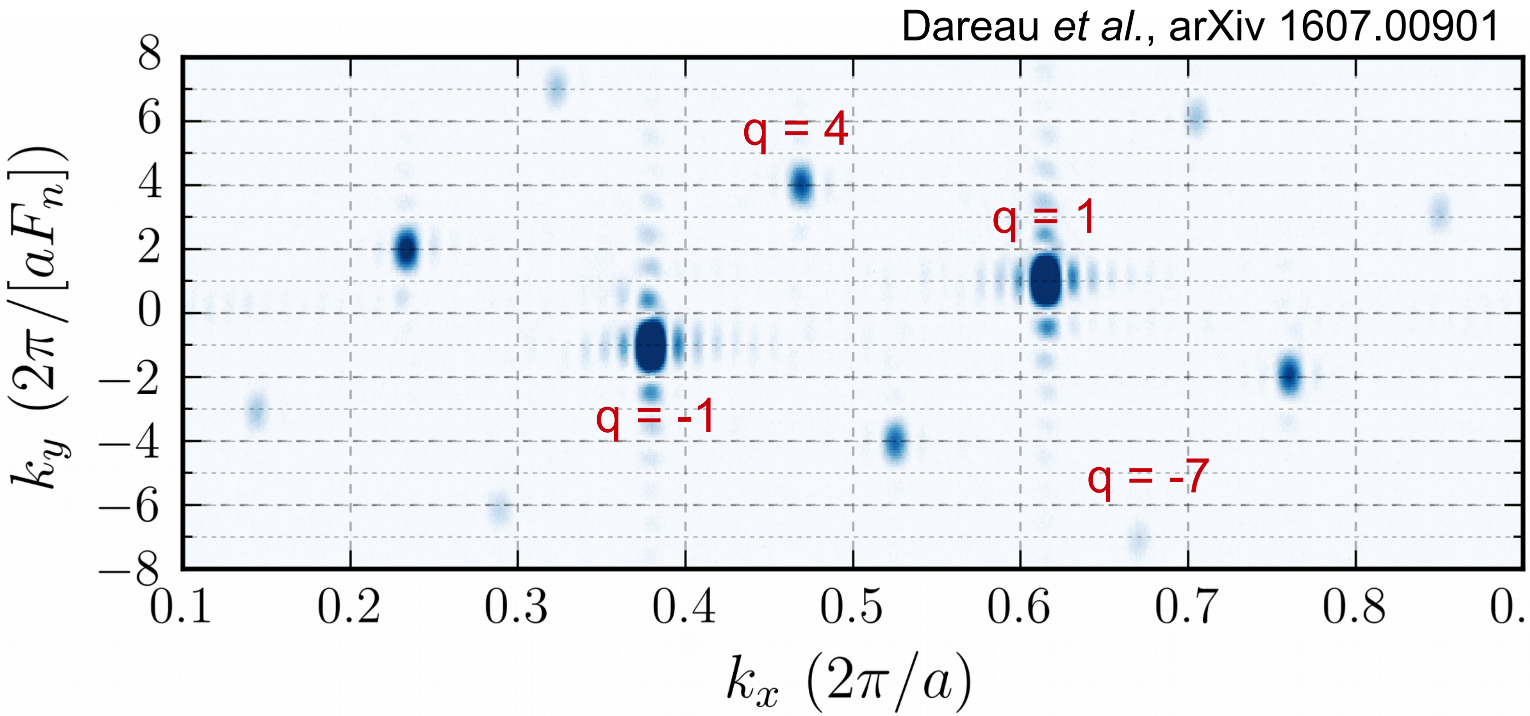


$$I(k_x, k_y) = \left| \sum_\phi \mathcal{A}_0(k_x) e^{i k_y a \frac{F_n \phi}{2\pi}} e^{-iq\phi} \right|^2 = |\mathcal{A}_0(k_x)|^2 \left| \sum_\phi e^{i k_y a \frac{F_n \phi}{2\pi}} e^{-iq\phi} \right|^2$$

peaks located at same k_x as before

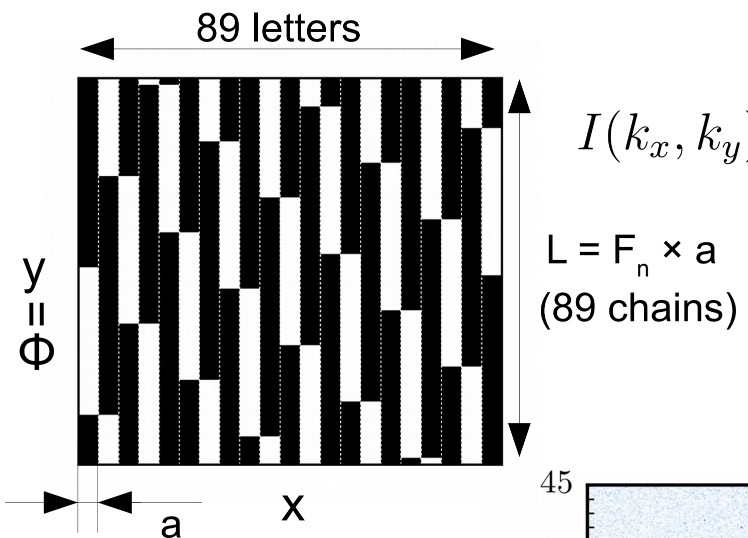
peak for $k_y = q \times \frac{2\pi}{aF_n}$

Peak position along y is proportional to the Chern number



Optical diffraction by a Fibonacci chain

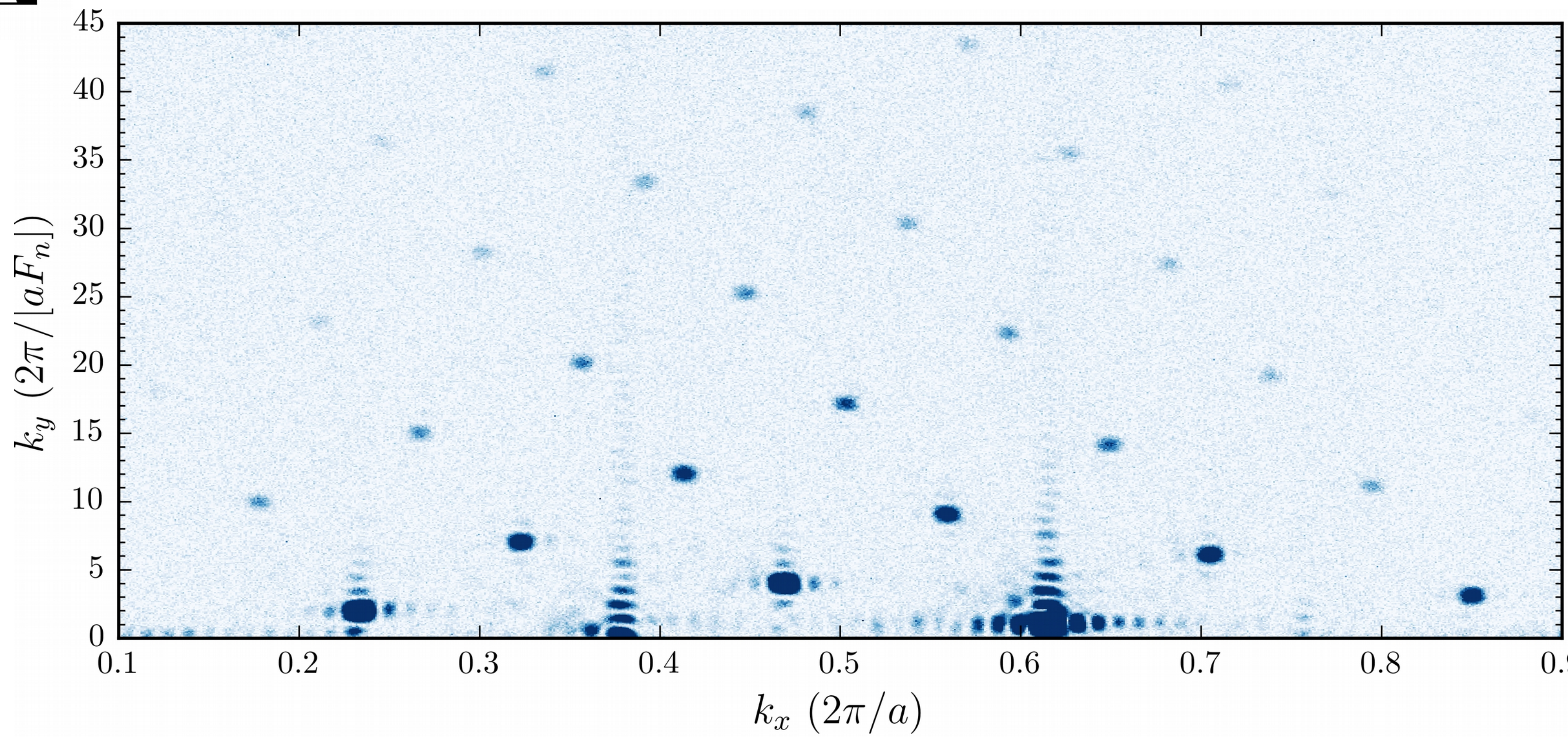
Diffraction from 2D (x,Φ) pattern



$$I(k_x, k_y) = \left| \sum_{\phi} \mathcal{A}_0(k_x) e^{ik_y a \frac{F_n \phi}{2\pi}} e^{-iq\phi} \right|^2 = |\mathcal{A}_0(k_x)|^2 \left| \sum_{\phi} e^{ik_y a \frac{F_n \phi}{2\pi}} e^{-iq\phi} \right|^2$$

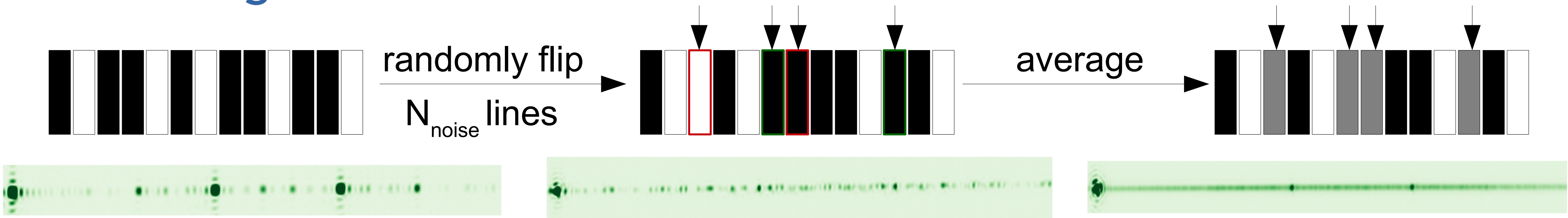
peaks located at same k_x as before

peak for $k_y = q \times \frac{2\pi}{aF_n}$



Optical diffraction by a Fibonacci chain

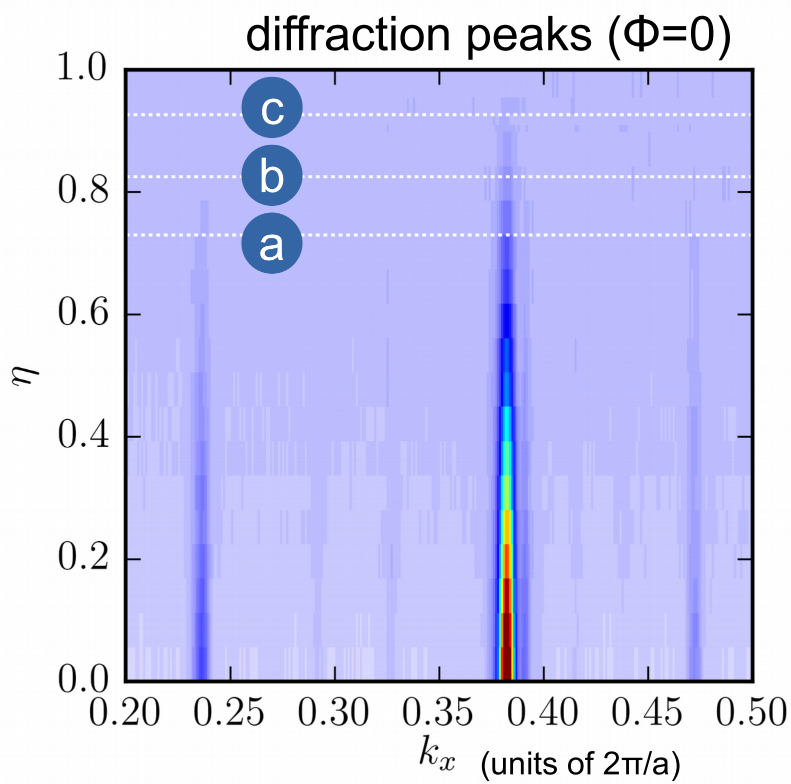
Testing robustness : effect of noise

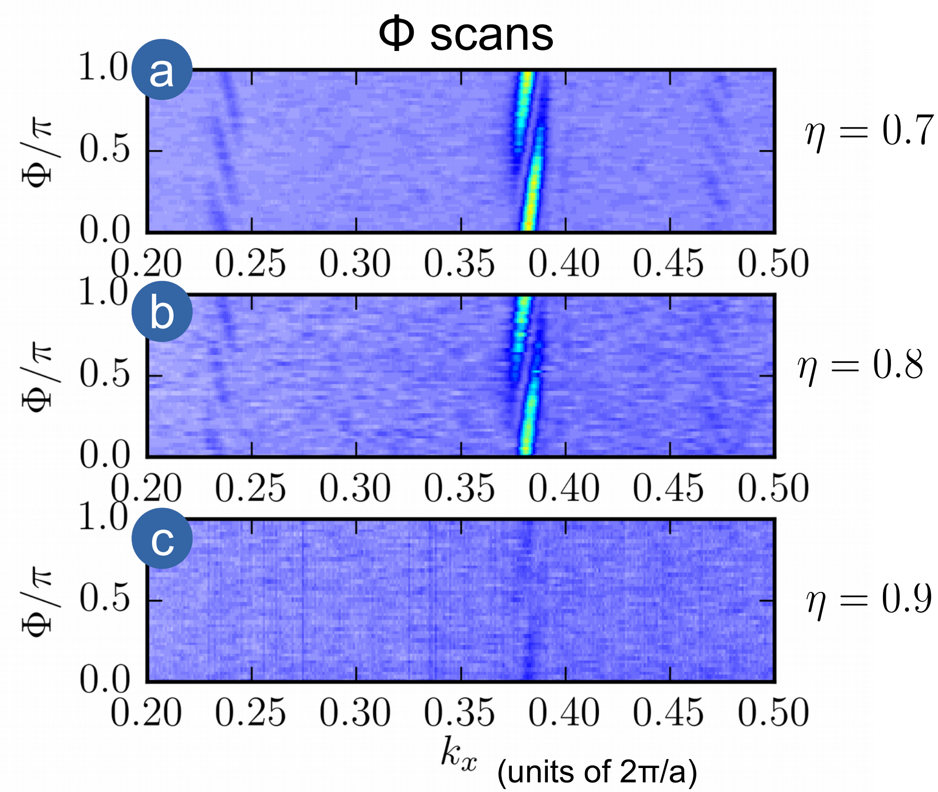


number of "noisy" lines

$$\eta = \frac{N_{\text{noise}}}{N_{\text{tot}}}$$

total number of lines



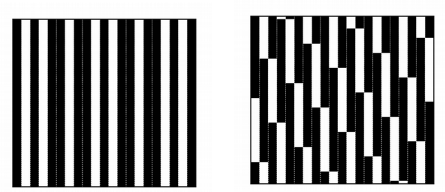


Hole crossing visible even for weak peak signal
(and number of crossings unchanged)

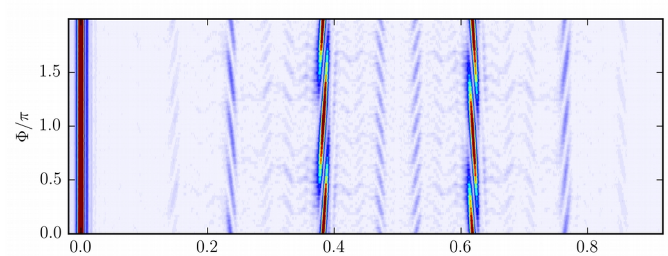
Conclusion and outlook

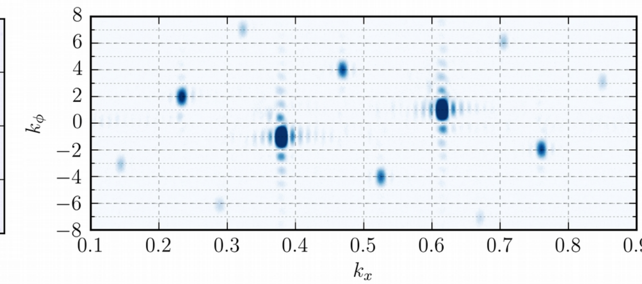
Experimental measurements

Diffraction on a optical 1D Fibonacci grating or a 2D set of Fibonacci chains



Reveals underlying topological properties of Fibonacci quasicrystals





→ Stresses the importance of the "phason" degree of freedom

Kraus *et al.*, PRL (2012), Levy *et al.*, arXiV (2015)

How to extend this method ?

→ Directly applicable to any quasicrystal generated with the "Cut & Project" method

→ Study effect of "phason" on 2D quasiperiodic tilings ?

→ Matter-waves diffraction / propagation in 1D quasiperiodic potential

DMD can be used to project the grating on an gas of cold atoms

Direct measurement of Chern numbers in the diffraction pattern of a Fibonacci chain.

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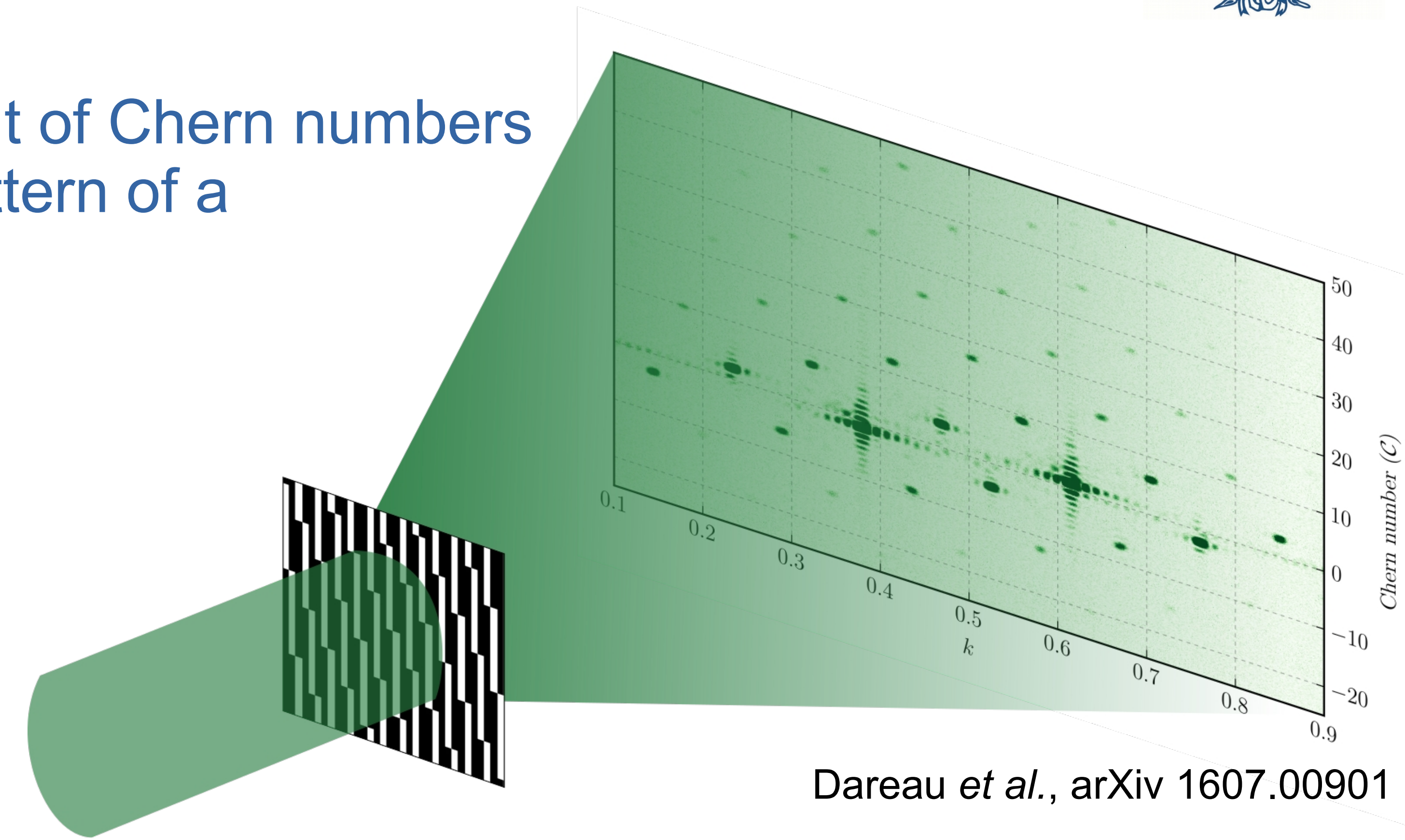

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(*) Technion Israel Institute of Technology, Department of Physics (Israel)

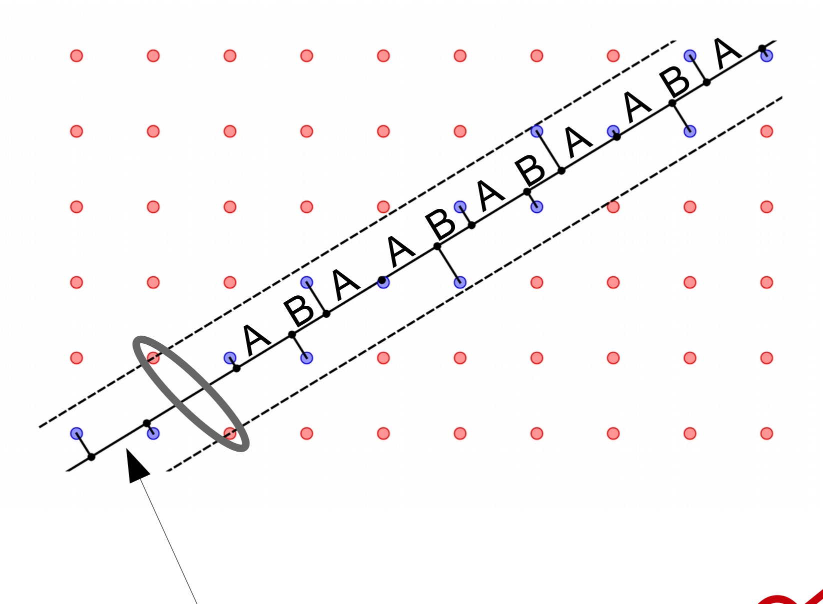
Phason degree of freedom

$$\tau = \frac{1+\sqrt{5}}{2}$$

- cut and project (C&P)
- characteristic function

Kraus *et al.*, PRL (2012)

$$\begin{cases} S_n = [\chi_1 \chi_2 \cdots \chi_n] \\ \chi_n = \text{sign}\left[\cos(2\pi n \tau^{-1} + \phi) - \cos(\pi \tau^{-1})\right] \end{cases}$$



line slope : $y = x\tau^{-1} + b$

additional degree of freedom = **“phason”**

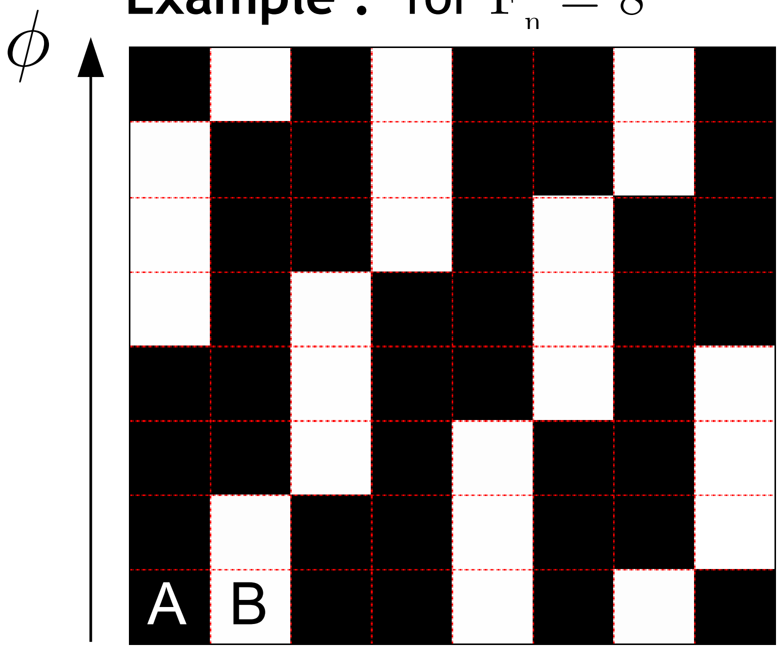
Fibonacci Chain

Effect of the phason ?

For a finite chain of length F_n ⟶ Scanning Φ over 2π generates F_n different configurations

NB : The generated configurations are segments of the infinite chain

Example : for $F_n = 8$



Infinite chain :

ABAABABAABAABABAABABA…

$\phi = 2 \times (2\pi/F_n)$ ⟶ AABABAAB

$\phi = 1 \times (2\pi/F_n)$ ⟶ ABAABAAB

$\phi = 0 \times (2\pi/F_n)$ ⟶ ABAABABA

➡ Spatial shift $\Delta X = [(-1)^n F_{n-1} + j F_n] \times (\phi F_n / 2\pi)$ $(F_{n-1} = 5)$

$j \in \mathbb{Z}$